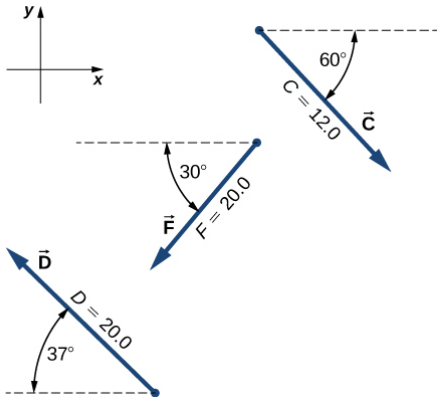
Problem 53

Given the vectors in the preceding figure, find vector $\vec{\mathbf{R}}$ that solves equations (a) $\vec{\mathbf{D}} + \vec{\mathbf{R}} = \vec{\mathbf{F}}$ and (b) $\vec{\mathbf{C}} - 2\vec{\mathbf{D}} + 5\vec{\mathbf{R}} = 3\vec{\mathbf{F}}$. Assume the $+x$-axis is horizontal to the right.

Solution

The relevant vectors are shown below.



Note that when an angle above or below a horizontal is given, multiplying the magnitude by the cosine of the angle gives the horizontal component, and multiplying the magnitude by the sine of the angle gives the vertical component.

Part (a)

The given vector equation is

$$\vec{\mathbf{D}} + \vec{\mathbf{R}} = \vec{\mathbf{F}}.$$

Solve for $\vec{\mathbf{R}}$ by subtracting both sides by $\vec{\mathbf{D}}$.

$$\begin{aligned}\vec{\mathbf{R}} &= \vec{\mathbf{F}} - \vec{\mathbf{D}} \\ &= (-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}}) - (-20.0\cos 37^\circ\,\hat{\mathbf{i}} + 20.0\sin 37^\circ\,\hat{\mathbf{j}}) \\ &= (-20.0\cos 30^\circ + 20.0\cos 37^\circ)\hat{\mathbf{i}} + (-20.0\sin 30^\circ - 20.0\sin 37^\circ)\hat{\mathbf{j}} \\ &\approx -1.35\hat{\mathbf{i}} - 22.0\hat{\mathbf{j}}\end{aligned}$$

Part (b)

The given vector equation is

$$\vec{\mathbf{C}} - 2\vec{\mathbf{D}} + 5\vec{\mathbf{R}} = 3\vec{\mathbf{F}}.$$

Subtract both sides by $\vec{\mathbf{C}}$.

$$-2\vec{\mathbf{D}} + 5\vec{\mathbf{R}} = 3\vec{\mathbf{F}} - \vec{\mathbf{C}}$$

Add both sides by $2\vec{\mathbf{D}}$.

$$5\vec{\mathbf{R}} = 3\vec{\mathbf{F}} - \vec{\mathbf{C}} + 2\vec{\mathbf{D}}$$

Divide both sides by 5 to get $\vec{\mathbf{R}}$.

$$\vec{\mathbf{R}} = \frac{3}{5}\vec{\mathbf{F}} - \frac{1}{5}\vec{\mathbf{C}} + \frac{2}{5}\vec{\mathbf{D}}$$

$$= \frac{3}{5}(-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}}) - \frac{1}{5}(12.0\cos 60^\circ\,\hat{\mathbf{i}} - 12.0\sin 60^\circ\,\hat{\mathbf{j}}) + \frac{2}{5}(-20.0\cos 37^\circ\,\hat{\mathbf{i}} + 20.0\sin 37^\circ\,\hat{\mathbf{j}})$$

$$= (-12.0\cos 30^\circ - 2.40\cos 60^\circ - 8.0\cos 37^\circ)\hat{\mathbf{i}} + (-12.0\sin 30^\circ + 2.40\sin 60^\circ + 8.0\sin 37^\circ)\hat{\mathbf{j}}$$

$$\approx -18.0\hat{\mathbf{i}} + 0.893\hat{\mathbf{j}}$$